Influence of helicity on the Kolmogorov regime in fully developed turbulence

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The influence of helicity on the stability of the Kolmogorov scaling regime in fully developed turbulence in space dimension $d=3$ based on the stochastic Navier-Stokes equation with the self-similar Gaussian random stirring force δ-correlated in time and with the correlator proportional to $k^{4-d-2\varepsilon}$ is investigated by the field-theoretic renormalization-group technique within two-loop approximation. The two-loop renormalization constant, the β function, and the coordinate of the fixed point are found as functions of the helicity parameter. It is shown that the presence of helicity in the system does not destroy the stability of the Kolmogorov scaling regime.

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I. INTRODUCTION

It can be said without doubt that as a starting point of the modern investigations of the phenomenon known as fully developed turbulence can be considered the celebrated works of Kolmogorov published in 1941 [1]. Nowadays, the corresponding theory is known as the phenomenological Kolmogorov-Obukhov (KO) theory, and the main conclusions of this theory are subjects of the famous Kolmogorov hypotheses [2–4].

For definiteness, consider as an example the single-time structure functions of the velocity field

$$S_N(r) = \langle [v_r(t,\mathbf{x}) - v_r(t,\mathbf{x}')]^N \rangle, \quad r = |\mathbf{x}-\mathbf{x}'|, \qquad (1)$$

where v_r denotes the component of the velocity field directed along the vector $\mathbf{r}=\mathbf{x}-\mathbf{x}'$. According to the first and the second Kolmogorov hypotheses the structure functions [Eq. (1)] are independent of both the external (integral) scale L and the internal (viscous) scale l within the so-called inertial range ($l \ll r \ll L$). Then, using simple dimensional analysis one comes to the following scale-invariant result

$$S_N(r) = \text{const} \times (\bar{\epsilon} r)^{N/3}, \qquad (2)$$

where $\bar{\epsilon}$ is the mean dissipation rate.

On the other hand, both experimental and theoretical studies show the existence of deviations from the predictions of the KO theory, namely, the dependence of the correlation functions on the integral scale L is detected in contradiction with the first Kolmogorov hypothesis [2,4–7]. Such deviations, referred to as anomalous or nondimensional scaling, manifest themselves in a singular dependence of the correlation functions on the distances and the integral scale L and, as a consequence, the simple scaling representation [Eq. (2)] must be replaced by the following one:

$$S_N(r) = (\bar{\epsilon} r)^{N/3} R_N(r/L), \qquad (3)$$

with some unknown scaling functions R_N. The assumption that they have powerlike asymptotic behavior in the region $r \ll L$ in the form

$$R_N(r/L) \sim (r/L)^{q_N}, \qquad (4)$$

with singular dependence on L in the limit $L \to \infty$ and nonlinearity of the exponents q_N as functions of N is called "anomalous scaling" and it is explained by the existence of strong developed fluctuations of the dissipative rate (intermittency) [2–6].

Thus, the main aim of the theory of fully developed turbulence is to describe and verify the basic conclusions of the KO phenomenological theory and to identify possible deviations from this theory on the basis of a microscopic model.

As the most convenient microscopic model of fully developed turbulence is traditionally considered the stochastic Navier-Stokes equation driven by an external random force that simulates the energy pumping into the system by the large-scale modes to maintain steady state [2–4].

On the other hand, it is well known that an effective and powerful method for investigation of self-similar scaling behavior is the renormalization-group (RG) technique [8–10], which can be also used in the theory of fully developed turbulence. In the framework of the most formal variant of the RG technique, namely, in the so-called field-theoretic RG approach [10–14], the analysis of the scaling properties of fully developed turbulence can be divided into two main stages. On the first stage the multiplicative renormalizability of the corresponding field-theoretic model is demonstrated, and the differential RG equations for correlation functions are obtained [e.g., for the aforementioned single-time structure functions given in Eq. (1)]. The asymptotic scaling behavior of the latter on their argument (r/l) for $r \gg l$ and any fixed (r/L) is given by the infrared stable fixed point of those equations with critical dimensions which are in accordance with those predicted by the KO theory. On the second stage, the behavior of the scaling functions on the infrared argument (r/L) for $r \ll L$ is found from the operator product expansion (OPE), where the crucial role is played by the composite operators with negative critical dimensions. However, it must be stressed that the second stage of the analysis for the velocity field within the fully developed turbulence is still an open problem as a result of the fact that dangerous operators enter the corresponding OPE in the form of infinite families with spectra of their critical dimensions unbounded from below. Thus, the nontrivial problem of their summation arises [10,13–15]. Nevertheless, the great progress has been achieved in this direction for the correlation functions of passive scalar or vector field advected by the velocity field with given Gaussian or non-Gaussian statistics (see, e.g., Refs. [6,16,17] and references cited therein).

In what follows, however, we shall concentrate only on the first stage of the RG analysis, and our aim will be to find possible influence of the broken spatial parity (helicity) of the turbulent environment on the stability of the Kolmogorov scaling regime. This question is not interesting only by itself but is also important for further investigations of the physical systems, where helicity can play central role. Let us discuss it briefly.

Helicity is defined as the scalar product of velocity and vorticity, and its nonzero value expresses mirror symmetry breaking of the turbulent flow. It represents one of the most important characteristics of large-scale motions and can be observed in various natural (such as large air vortices in the atmosphere) and technical flows [18–23].

Despite this fact the role of the helicity in hydrodynamical turbulence is not completely clarified up to now. Hence, let us discuss some known facts. The Navier-Stokes equations conserve kinetic energy and helicity in the inviscid limit. The presence of two quadratic invariants leads to the possibility of appearance of double cascade. This means that cascades of energy and helicity take place in different ranges of wave numbers analogously to the two-dimensional turbulence and the helicity cascade appears concurrently to the energy cascade in the direction of small scales [24,25]. In particular, the helicity cascade is closely connected with the existence of the exact relation between triple and double correlations of velocity known as 2/15 law analogously to the 4/5 Kolmogorov law [26]. Corresponding to [24], the aforementioned scenarios of turbulent cascades differ from each other by spectral scaling. Theoretical arguments given by Kraichnan [27] and results of numerical calculations of the Navier-Stokes equations [28–30] support the scenario of concurrent cascades. The appearance of helicity in turbulent system leads to constraint of nonlinear cascade to small scales. This phenomenon was first demonstrated by Kraichnan [27] within the modeling problem of statistically equilibrium spectra and later in numerical experiments.

On the other hand, the theoretical confirmation of the known fact that the stability of the scaling regime under influence of helicity is not disrupt was not done yet in the framework of a microscopic model what is the subject of the present paper. The result is also important for further fundamental theoretical investigations of physical systems, where helicity can play an important role. Brief discussion of such problems follows.

Maybe the most interesting example of a problem with nontrivial role of helicity is the turbulent dynamo problem: the generation of a large-scale stable magnetic field by the turbulent motion of a conductive fluid [31–33]. Within the field-theoretic RG approach this problem was studied to the first order in perturbation theory (one-loop approximation) in Refs. [34,35], but the stability of the corresponding kinetic scaling regime of the stochastic magnetohydrodynamic turbulence (which is related to the turbulent dynamo) does not depend on helicity at this approximation level. It is related to the fact that helicity is a pseudoscalar quantity, hence, its influence appears only in quadratic and higher terms of perturbation theory or in combination with other pseudoscalar quantities (e.g., large-scale helicity). Thus, for deeper understanding of the problem, it is necessary to go beyond the first-order approximation.

Another important question is related to the possible dependence of the anomalous exponents of the single-time structure functions of a passive scalar field advected by the given statistics of the velocity field on spatial parity violation. Within the models with Gaussian statistics of the velocity field the corresponding anomalous exponents do not depend on helicity as was shown in Ref. [36,37]. The open question is: is it related to the Gaussianity of the velocity field or will it be also true for more realistic velocity field driven by stochastic Navier-Stokes equation? That is, is there nontrivial helicity corrections to the anomalous dimensions found in Ref. [38]? As the first step, again, two-loop analysis of the stability of the Kolmogorov regime as a function of helicity is needed.

Last but not least, the dependence of the Kolmogorov constant and the inverse Prandtl number on helicity parameter is also a rather interesting question (details see in Refs. [15,39], respectively, where the corresponding two-loop calculations without helicity were done).

Thus, in what follows, we shall analyze the stochastic Navier-Stokes equation driven by helical random force using the field-theoretic renormalization group in two-loop approximation, and we shall find the answer to the question whether the presence of helicity in the system can lead to the destruction of the Kolmogorov scaling regime. The corresponding nonhelical model was investigated in Ref. [15].

The main result of the paper is the conclusion that even though the two-loop helical contribution to the final result is of the same order as the nonhelical contribution, and at the same time, it has the opposite sign, nevertheless, the stability of the Kolmogorov scaling regime is not disturbed.

The paper is organized as follows. In Sec. II, the model is defined and the field-theoretic formulation of the model is given. In Sec. III, we perform the ultraviolet (UV) renormalization of the model, and the two-loop renormalization constant is calculated. In Sec. IV, stability of the Kolmogorov scaling regime as function of a helicity parameter is discussed. Obtained results are briefly reviewed and discussed in Sec. V.

II. MODEL AND ITS FIELD-THEORETIC FORMULATION

A. Model of fully developed turbulence with helicity

Traditionally, description of fully developed turbulence of an incompressible fluid at the microscopic level is based on the stochastic Navier-Stokes equation with a random stirring force

$$\partial_t \mathbf{v} = \nu_0 \Delta \mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{v} - \nabla P + \mathbf{f}, \qquad (5)$$

where $\mathbf{v}(t,\mathbf{x})$ is the transverse (due to the incompressibility) vector velocity field, $P(t,\mathbf{x})$ is a pressure, ν_0 is the kinematic viscosity coefficient (in what follows, a subscript 0 will denote bare parameters of the unrenormalized theory), and $\Delta = \nabla^2$ is the Laplace operator. The transverse random force per unit mass $\mathbf{f}=\mathbf{f}(t,\mathbf{x})$ in Eq. (5) simulates the energy pumping into the system on large scales to maintain the steady state. We assume that its statistics is Gaussian with zero mean and pair-correlation function

$$D_{ij}(x,x') \equiv \langle f_i(x) f_j(x') \rangle$$
$$= \delta(t-t') \int \frac{d\mathbf{k}}{(2\pi)^d} R_{ij}(\mathbf{k}) D_0 k^{4-d-2\varepsilon} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}, \quad (6)$$

where $x \equiv (t,\mathbf{x})$, d denotes the spatial dimension of the system, $R_{ij}(\mathbf{k})$ is a transverse projector, $D_0 \equiv g_0 \nu_0^3 > 0$ is the positive amplitude, and the physical value of formally small parameter $0 < \varepsilon \leq 2$ is $\varepsilon = 2$. It plays an analogous role as the parameter $\epsilon = 4-d$ in the theory of critical behavior, and the introduced parameter g_0 plays the role of the coupling constant of the model. In addition, g_0 is a formal small parameter of the ordinary perturbation theory, and it is related to the characteristic UV momentum scale Λ (or inner length $l \sim \Lambda^{-1}$) by the following relation:

$$g_0 \simeq \Lambda^{2\varepsilon}. \quad (7)$$

The Galilean invariance of the stochastic model [Eqs. (5) and (6)] is guaranteed by the time decorrelation of the random force.

The transition to a helical fluid corresponds to the giving up of conservation of spatial parity, and technically this is expressed by the fact that the correlation function is specified in the form of a mixture of a tensor and a pseudotensor. In our isotropic case, it means that the transverse projector R_{ij} can be divided into two parts;

$$R_{ij}(\mathbf{k}) = P_{ij}(\mathbf{k}) + H_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2 + i\rho\varepsilon_{ijl} k_l/k, \quad (8)$$

which consists of the nonhelical standard transverse projector $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2$ and $H_{ij}(\mathbf{k}) = i\rho\varepsilon_{ijl} k_l/k$, which represents the presence of helicity in the flow. Here, ε_{ijl} is the Levi-Civita's completely antisymmetric tensor of rank 3, and the real parameter of helicity, ρ, characterizes the amount of helicity. Due to the requirement of positive definiteness of the correlation function the absolute value of ρ must be in the interval $|\rho| \in [0,1]$. Physically, the nonzero helical part expresses the existence of nonzero correlations $\langle \mathbf{v} \cdot \mathrm{rot}\, \mathbf{v} \rangle$ in the system.

The correlation function in Eq. (6) is chosen in the form which, on one hand, is suitable for description of the real infrared energy pumping to the system [for $\varepsilon \to 2$ the function $D_0 k^{4-d-2\varepsilon}$ is proportional to $\delta(\mathbf{k})$ for appropriate choice of the amplitude factor D_0, which corresponds to the injection of energy to the system through interaction with the largest turbulent eddies], and on the other hand, its powerlike form gives possibility to apply the RG technique for analysis of the problem [10,14,15].

B. Field-theoretic formulation of the model

Using the well-known theorem [40] the stochastic problem [Eqs. (5) and (6)] is equivalent to the field-theoretic model of the transverse fields $\mathbf{v}$ and $\mathbf{v}'$ with action functional

$$S(\mathbf{v},\mathbf{v}') = \frac{1}{2} \int dx_1 dx_2 v_i'(x_1) D_{ij}(x_1;x_2) v_j'(x_2)$$
$$+ \int dx \mathbf{v}' \cdot [-\partial_t \mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{v} + \nu_0 \Delta \mathbf{v}], \quad (9)$$

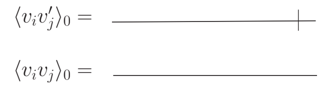

FIG. 1. Graphical representation of the propagators of the model.

where D_{ij} denotes a random force correlator [Eq. (6)], $dx = dt d^d\mathbf{x}$, and the required summation over dummy indices is assumed.

Model (9) corresponds to a standard Feynman diagrammatic perturbation theory with bare propagators (in frequency-momentum representation)

$$\langle v_i v_j' \rangle_0 = \langle v_i' v_j \rangle_0^* = \frac{P_{ij}}{-i\omega + \nu_0 k^2}, \quad (10)$$

$$\langle v_i v_j \rangle_0 = \frac{g_0 \nu_0^3 k^{4-d-2\varepsilon} R_{ij}(\mathbf{k})}{(-i\omega + \nu_0 k^2)(i\omega + \nu_0 k^2)}. \quad (11)$$

Their graphical representation is presented in Fig. 1. The triple (interaction) vertex,

$$-v_i'(v_j \partial_j) v_i = v_i' V_{ijl} v_j v_l/2, \quad V_{ijl} = i(k_j \delta_{il} + k_l \delta_{ij}), \quad (12)$$

is shown in Fig. 2, where the momentum $\mathbf{k}$ is flowing into the vertex via the auxiliary field $\mathbf{v}'$.

The advantage of the formulation of the stochastic problem given by Eqs. (5) and (6) through action functional (9) is that it makes it possible to apply the well-defined field-theoretic means, e.g., the RG technique, to analyze the problem and, at the same time, the statistical averages of random quantities in the stochastic problem are replaced with the corresponding functional averages with weight $\exp S(\mathbf{v},\mathbf{v}')$. Thus, the generating functionals of the total Green's functions $G(\mathbf{A},\mathbf{A}')$ and connected Green's functions $W(\mathbf{A},\mathbf{A}')$ are defined by the functional integral

$$G(\mathbf{A},\mathbf{A}') = e^{W(\mathbf{A},\mathbf{A}')}$$
$$= \int \mathcal{D}\mathbf{v}\mathcal{D}\mathbf{v}' \exp\left[S(\mathbf{v},\mathbf{v}') + \int dx (A_i v_i + A_i' v_i') \right], \quad (13)$$

where $\mathbf{A}$ and $\mathbf{A}'$ are the arbitrary sources for the fields $\mathbf{v}$ and $\mathbf{v}'$, respectively, and $\mathcal{D}\mathbf{v}\mathcal{D}\mathbf{v}'$ denotes the measure of functional integration.

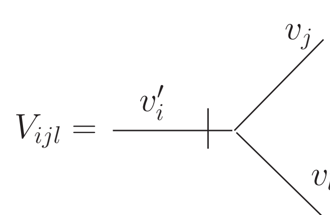

FIG. 2. The interaction vertex of the model.

III. RENORMALIZATION-GROUP ANALYSIS

A. General discussion

The field-theoretic model described by action functional (5) belongs to the class of the so-called two-scaled models [10,13,14], i.e., to the class of models for which the canonical dimension of some quantity Q is given by two numbers: the momentum dimension d_Q^k and the frequency dimension d_Q^ω. Hence, the dimensions of all quantities can be found by using the requirement that each term of the action functional must be dimensionless separately with respect to the momentum and frequency together with the standard definitions (normalization conditions) $d_k^k=-d_x^k=d_\omega^\omega=-d_t^\omega=1$. The total canonical dimension d_Q is then defined as $d_Q=d_Q^k+2d_Q^\omega$, and it plays the same role in the renormalization theory of our dynamical model as the simple momentum dimension does in static models.

The RG analysis of a field-theoretic model is based on the analysis of the UV divergences of the model. It is well known that the superficial divergences can be present only in the one-irreducible Green's functions for which the corresponding total canonical dimensions are a non-negative integer. Detailed dimensional analysis of action (9) for space dimensions $d>2$ [12] shows that the model is logarithmic at $\varepsilon=0$, i.e., the coupling constant g_0 is dimensionless at this point, and that the superficial UV divergences can be present only in the one-irreducible functions $\langle v_i' v_j\rangle_{1-ir}$ and $\langle v_i' v_j v_l\rangle_{1-ir}$ with the following possible counterterms: $v_i'\Delta v_j$, $v_i'\partial_t v_j$, and $v_i'(v_j\partial_j)v_l$. However, using integration by parts it is possible to move the derivative from the field v onto the field v' in the interaction vertex [Eq. (12)]. It means that all potential counterterms to the above mentioned one-irreducible functions must contain at least one spatial derivative. Therefore, the structure $v_i'\partial_t v_j$ cannot play the role of a counterterm. This fact excludes also the structure $v_i'(v_j\partial_j)v_l$ as potential counterterm because of the Galilean symmetry of action (9). This symmetry requires that the structures $v_i'\partial_t v_j$ and $v_i'(v_j\partial_j)v_l$ can enter the counterterms only in the Galilean invariant form, namely, $v_i'[\partial_t+(v_j\partial_j)]v_l$. Therefore, the function $\langle v_i' v_j v_l\rangle_{1-ir}$ must be also UV finite and the only one-irreducible function which will possess UV divergences is $\langle v_i' v_j\rangle_{1-ir}$. In the minimal subtraction (MS) scheme [9], which is used in what follows, the UV divergences have the form of poles in ε and can be removed multiplicatively by the only counterterm $v_i'\Delta v_j$. It can be explicitly expressed in the multiplicative renormalization of the bare parameters g_0 and ν_0 without renormalization of the others parameters and fields, namely,

$$\nu_0 = \nu Z_\nu, \quad g_0 = g\mu^{2\varepsilon} Z_g, \tag{14}$$

where the dimensionless parameters g and ν are the renormalized counterparts of the corresponding bare ones, μ is the renormalization mass (a scale setting parameter), an artifact of the dimensional regularization, and $Z_i=Z_i(g,\rho,d,\varepsilon)$, $i=g,\nu$ are the so-called renormalization constants.

The renormalized action functional has the following form:

$$S^R(\mathbf{v},\mathbf{v}') = \frac{1}{2}\int dx_1 dx_2 v_i'(x_1) D_{ij}(x_1;x_2) v_j'(x_2) + \int dx \mathbf{v}'\cdot[-\partial_t - (\mathbf{v}\cdot\partial) + \nu Z_\nu \Delta]\mathbf{v}, \tag{15}$$

and by comparison of the renormalized action [Eq. (15)] with definitions of the renormalization constants Z_i, $i=g,\nu$ together with the fact that $D_0=\mu^{2\varepsilon}D$ (or equivalently $g_0\nu_0^3=g\nu^3\mu^{2\varepsilon}$), one obtains relation between Z_g and Z_ν, namely,

$$Z_g = Z_\nu^{-3}. \tag{16}$$

Thus, one is left with the only independent renormalization constant Z_ν, and its explicit form in the MS scheme is

$$Z_\nu(g,\rho,d,\varepsilon) = 1 + \sum_{n=1}^{\infty} g^n \sum_{j=1}^{n} \frac{z_{nj}(\rho,d)}{\varepsilon^j}, \tag{17}$$

where the coefficients z_{nj} are independent of ε.

In the nonhelical situation ($\rho=0$) the expansion of the renormalization constant Z_ν is known up to second order in g (two-loop approximation), i.e., the explicit form of the coefficients z_{11}, z_{21}, and z_{22} was already calculated. The simplest one-loop result z_{11} was calculated in Ref. [12], and it reads

$$z_{11} = -\frac{S_d}{(2\pi)^d}\frac{(d-1)}{8(d+2)}, \tag{18}$$

where S_d denotes the surface area of the d-dimensional unit sphere defined as

$$S_d \equiv \frac{2\pi^{d/2}}{\Gamma(d/2)}, \tag{19}$$

and $\Gamma(x)$ is Euler's Gamma function. On the other hand, the two-loop corrections z_{21} and z_{22} were found by authors of the paper [15]. The main aim of the present work is to find the explicit dependence of the coefficient z_{21} on the helicity parameter ρ (the coefficients z_{11} and z_{22} are independent of ρ) and to analyze the corresponding consequences on the Kolmogorov scaling regime.

First, let us briefly discuss the consequences of the renormalization procedure. The fact that fields $\mathbf{v}$ and $\mathbf{v}'$ are not renormalized means that the renormalized correlation functions $W^R=\langle v'\cdots v' v\cdots v\rangle^R$ are equal to their unrenormalized counterparts $W=\langle v'\cdots v' v\cdots v\rangle$, and the only difference is in the choice of variables (renormalized or unrenormalized) and in the corresponding perturbation expansion (in g or g_0), i.e.,

$$W^R(g,\nu,\mu,\ldots) = W(g_0,\nu_0,\ldots), \tag{20}$$

where the dots stand for other arguments which are untouched by renormalization, e.g., the helicity parameter, coordinates, and times. Using the fact that unrenormalized correlation functions are independent of the scale-setting parameter μ one can apply the differential operator $\mathcal{D}_\mu \equiv \mu\partial_\mu$ on both sides of Eq. (20) which leads to the basic differential RG equation

$$[\mu\partial_\mu + \beta_g(g)\partial_g - \gamma_\nu(g)\nu\partial_\nu]W^R(g,\nu,\mu,\ldots) = 0, \tag{21}$$

where the so-called RG functions (the β and γ functions) are given as

$$\beta_g \equiv \mu\partial_\mu g = g(-2\varepsilon + 3\gamma_\nu), \quad \gamma_\nu \equiv \mu\partial_\mu \ln Z_\nu, \tag{22}$$

and the relation $\gamma_g = -3\gamma_\nu$ was used which follows from Eq. (16).

The IR asymptotic scaling behavior of the correlation functions of the model is driven by the IR stable fixed point of the RG equations. The simplest way to find the coordinates of a fixed point is using the requirement of the vanishing of β_g at this point, i.e.,

$$\beta_g(g_*) = g_*(-2\varepsilon + 3\gamma_\nu^*) = 0, \tag{23}$$

where fixed-point values of all quantities are denoted by the star. It immediately leads to the exact value for the anomalous dimension γ_ν at the fixed point g_*, namely,

$$\gamma_\nu^* = \frac{2\varepsilon}{3}. \tag{24}$$

The form of the β_g in Eq. (22) does not depend on order of the perturbation expansion, i.e., it is perturbatively stable (it is exactly given at one-loop approximation within the perturbation theory without higher-loop corrections). Strictly speaking, it is given immediately by the renormalization procedure because for its determination even one-loop calculations of Z_ν are not needed.

Nevertheless, the explicit form of the renormalization constant Z_ν is needed for determination of the IR attraction of the fixed point because without the corresponding analysis we are not able to make the corresponding meaningful conclusion. In our case, the sufficient condition for IR stability of the fixed point is the positiveness of the first derivative of β_g with respect to g taken at the fixed point g_*:

$$\Omega \equiv \partial_g \beta_g|_* > 0. \tag{25}$$

If this condition is fulfilled then at the IR stable fixed point the differential RG Eq. (21) obtains form of the differential equation for a generalized homogenous function W^R with a solution which has a scaling form.

For example, as was already discussed in Sec. I, the existence of the stable IR fixed point leads to the following IR ($l \ll r$) scaling behavior (for physical value $\varepsilon=2$) [13,14]:

$$S_N(r) \sim r^{N/3} R_N(r/L), \tag{26}$$

of the single-time structure functions [Eq. (1)] with some scaling functions R_N which are not determined by the RG equations but can be studied by the OPE which leads to the so-called anomalous scaling (see Sec. I for details and the corresponding references cited therein).

However, as was already mentioned, the problem of the anomalous scaling is out of scope of the present work. In what follows, we shall concentrate only on the existence of the IR scaling behavior in the model and our aim is to investigate the influence of the helical energy pumping on the stability of the Kolmogorov regime. It means that the explicit dependence of Ω in Eq. (25) on the helicity parameter ρ must be found.

B. $Z_\nu(\rho)$ in two-loop approximation

To proceed further it is necessary to find the explicit form of the renormalization constant Z_ν. It is determined by the requirement that the one-irreducible Green's function $\langle v_i' v_j \rangle_{1-ir}$ must be UV finite when it is written in the renormalized variables, i.e., it must be free of poles in ε. On the other hand, the one-irreducible Green's function $\langle v_i' v_j \rangle_{1-ir}$ is related to the self-energy operator $\Sigma^{v'v}$ by the Dyson equation which can be written in the following convenient form:

$$\langle v_i' v_j \rangle_{1-ir} = (-i\omega + \nu_0 p^2)\delta_{ij} - \Sigma_{ij}^{v'v}(\omega,p). \tag{27}$$

Thus, Z_ν is found from requirement that the UV divergences are canceled in Eq. (27) after the substitution $\nu_0 = \nu Z_\nu$ and $g_0 = g\mu^{2\varepsilon} Z_g$ ($Z_g = Z_\nu^{-3}$). This determines Z_ν up to UV finite contribution which is fixed by the choice of the renormalization scheme. As was already mentioned, we work in the MS scheme with the standard form of the Z_ν as it is given in Eq. (17).

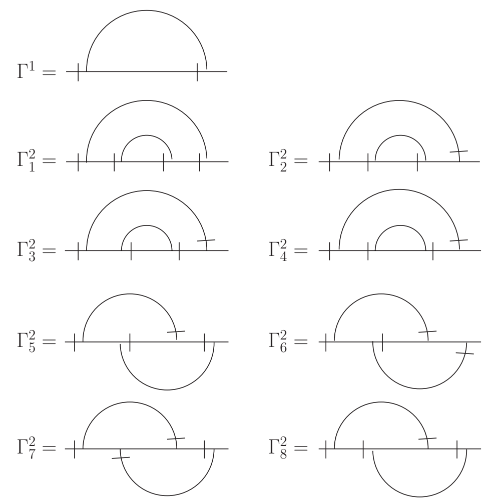

FIG. 3. The one-loop and two-loop diagrams that contribute to the self-energy operator $\Sigma_{ij}^{v'v}(\omega,p)$.

The self-energy operator $\Sigma_{ij}^{v'v}$ is given by the sum of singular parts of the corresponding one-irreducible diagrams. In two-loop approximation it reads (we omit the indexes i and j for simplicity)

$$\Sigma^{v'v} = \Gamma^1 + \Gamma^2 = \Gamma^1 + \sum_{l=1}^{8} s_l \Gamma_l^2, \tag{28}$$

where $s_l, l=1,\ldots,8$ represent the corresponding symmetry coefficients of the two-loop diagrams which are shown in Fig. 3. The analytic form of the singular part of the one-loop contribution Γ^1 is given as (in general $d>2$ case)

$$\Gamma^1 = -\frac{S_d}{(2\pi)^d}\frac{g\nu p^2 \delta_{ij}}{8\varepsilon}\left(\frac{\mu}{m}\right)^{2\varepsilon}\frac{d-1}{d+2}, \tag{29}$$

and does not depend on helicity. In Eq. (29), $m=1/L$ is an integral scale, and it is introduced to provide needed IR regularization (see, e.g., [15] for details). On the other hand, the two-loop contributions Γ_l^2 can be divided into the nonhelical and helical parts, namely,

$$\Gamma_l^2 = \Gamma_l^{2,0} + \rho^2 \Gamma_l^{2,\rho}, \quad l = 1, \ldots, 8. \tag{30}$$

The singular parts of the two-loop diagrams in Fig. 3 in the nonhelical case ($\rho=0$), i.e., the parts $\Gamma_l^{2,0}$ in Eq. (30), were calculated and shown explicitly in Ref. [15] in the form of double integrals. On the other hand, the determination of an explicit analytical form of the singular helical parts, $\Gamma_l^{2,\rho}$, is the main aim of the present work.

In principle a few ways exist to evaluate the two-loop diagrams Γ_l^2, $l=1,\ldots,8$. First of all, it is a method which was used in Ref. [15] where the nonhelical parts of the diagrams in Fig. 3 were calculated and the results were obtained in a compact integral form. We have used this method also for calculation of the helical parts of the diagrams, as well as we have recalculated the results obtained in Ref. [15] to confirm their correctness. On the other hand, to confirm the correctness of our expressions for the singular helical parts of the two-loop diagrams, we have also evaluated those using different method which was discussed as the first possible technique in Appendix A in Ref. [37]. These alternative expressions for the helical parts of the diagrams are shown in Appendix B and the corresponding comparison of the results is briefly discussed in Appendix C.

Thus, one possible explicit representation of the singular parts of the two-loop diagrams in Fig. 3 is the following integral form:

$$\Gamma_l^2 = \frac{g^2 \nu S_d}{16(2\pi)^{2d}} \frac{p^2 \delta_{ij}}{d(d+2)} \left(\frac{\mu}{m}\right)^{4\varepsilon} \frac{1}{\varepsilon} \left\{ \frac{d-1}{d+2} \frac{S_d}{\varepsilon} \frac{A_l}{4} + \frac{S_{d-1}}{d-1} \times \int_0^1 dx \frac{(1-x^2)^{(d-1)/2}}{x^2} [B_l - \rho^2 (d-2) C_l] \right\}, \tag{31}$$

where x is the cosine of the angle between two independent momenta $\mathbf{k}$ and $\mathbf{q}$ over which the integration is taken in two-loop case, i.e., $x=\mathbf{k}\cdot\mathbf{q}/(|\mathbf{k}||\mathbf{q}|)$ and

$$B_l = B_{l0} + B_{l1} X_1 + B_{l2} X_2, \tag{32}$$

$$C_l = C_{l1} Y_1 + C_{l2} Y_2, \tag{33}$$

with

$$X_1 = \arctan\left(\frac{1-x}{\sqrt{1-x^2}}\right) - \arctan\left(\frac{1+x}{\sqrt{1-x^2}}\right),$$

$$X_2 = \arctan\left(\frac{2-x}{\sqrt{4-x^2}}\right) - \arctan\left(\frac{2+x}{\sqrt{4-x^2}}\right),$$

$$Y_1 = \pi - \arctan\left(\frac{1-x}{\sqrt{1-x^2}}\right) - \arctan\left(\frac{1+x}{\sqrt{1-x^2}}\right),$$

$$Y_2 = \pi - \arctan\left(\frac{2-x}{\sqrt{4-x^2}}\right) - \arctan\left(\frac{2+x}{\sqrt{4-x^2}}\right),$$

and the explicit form of coefficients A_l, $B_{li}(i=0,1,2)$, and $C_{lj}(j=1,2)$ as functions of d and x are given in Appendix A. It is important to say that the above expressions for nonhelical parts of the diagrams are given in a slightly different form than the integral representation given in Ref. [15], namely, in Ref. [15] they are given in the form of double integrals, and our results are in the form of single integrals. Nevertheless, it can be shown by direct numerical analysis that both results are the same.

Finally, using the Dyson equation [Eq. (27)] and the explicit form of the one- and two-loop diagrams in the MS scheme given in Eqs. (29) and (31) the renormalization constant Z_ν in two-loop approximation is obtained in the following form (details see, e.g., in Ref. [15]):

$$Z_\nu = 1 + \frac{g}{\varepsilon} z_{11} + \frac{g^2}{\varepsilon}\left(\frac{z_{22}}{\varepsilon} + z_{21},\right) \tag{34}$$

where z_{11} is given in Eq. (18), z_{22} is simply defined by the coefficient z_{11} by the relation (see Ref. [15] for details)

$$z_{22} = -(z_{11})^2, \tag{35}$$

and

$$z_{21} = \frac{S_d S_{d-1}}{16(2\pi)^{2d}} \frac{1}{d(d-1)(d+2)} \int_0^1 dx \frac{(1-x^2)^{(d-1)/2}}{x^2} \times \sum_{l=1}^{8} s_l [B_l - \rho^2 (d-2) C_l], \tag{36}$$

where coefficients B_l and C_l are defined in Eqs. (32) and (33) and the explicit form of the vector $\mathbf{s}$ of the symmetry coefficients s_l, $l=1,\ldots,8$ is the following:

$$\mathbf{s} = (1,1,1,1/2,1,1,1,1). \tag{37}$$

The renormalization constant Z_ν in Eq. (34) is given in the general form as a function of d and ρ. Of course, strictly speaking, for the case with nonzero helicity ($\rho \neq 0$), only results taken directly in space dimension $d=3$ have the physical meaning (see Sec. IV).

IV. STABILITY OF THE KOLMOGOROV REGIME

The knowledge of the renormalization constant Z_ν in Eq. (34) in two-loop approximation [i.e., to the order $O(g^2)$] leads to the explicit two-loop expression for the anomalous dimension γ_ν defined in Eq. (22), namely,

$$\gamma_\nu \equiv \mu \partial_\mu \ln Z_\nu = -2(g z_{11} + 2 g^2 z_{21}), \tag{38}$$

and using Eq. (23) the two-loop coordinate g_* of the RG fixed point can be obtained in the form

$$g_* = -\frac{\varepsilon}{3 z_{11}} \left[1 + \frac{2\varepsilon z_{21}}{3 z_{11}^2}\right], \tag{39}$$

and its IR stability is driven by condition (25) with

$$\Omega = 2\varepsilon \left(1 - \frac{2\varepsilon z_{21}}{3 z_{11}^2}\right). \tag{40}$$

In the physical case with $d=3$ one has

$$z_{11} = -\frac{1}{40\pi^2}, \quad z_{21} = -0.00825 + 0.00557\rho^2, \tag{41}$$

and finally

$$\Omega = 2\varepsilon\left[1 - \frac{3200\pi^4}{3}(-0.00825 + 0.00557\rho^2)\varepsilon\right]. \quad (42)$$

The nonhelical part in z_{21} in Eq. (41) corresponds to the result obtained in Ref. [15], and the corresponding helical part represents result of the present paper. From this result, it is evident that even though the two-loop helical contribution is of the same order as the nonhelical one and, at the same time, has the opposite sign, nevertheless it cannot disturb the stability of the Kolmogorov scaling regime as it can be seen directly in Eq. (42). It means that the condition $\Omega > 0$ is fulfilled for $\varepsilon > 0$ and for all possible values of the helicity parameter ρ (we remind once more that the value of the parameter ρ must belong to the interval $[-1,1]$).

V. CONCLUSION

In present paper, we have investigated the influence of the broken spatial parity (helicity) on the stability of the Kolmogorov scaling regime in the turbulent system described by the stochastic Navier-Stokes equation driven by the Gaussian random force using the field-theoretic RG technique. The RG calculations are performed at two-loop approximation level, which is necessary to include the effects of helicity. The explicit dependence of the renormalization constant and the corresponding β and γ functions on the helicity parameter is found. It is shown that corresponding two-loop helical contribution to the first derivative of the β function taken at the fixed point has the opposite sign, i.e., it tends to destroy the stability of the scaling regime. However, the absolute value of the corresponding nonhelical contribution is larger than helical one for all possible values of the helicity parameter which leads to the fact that the Kolmogorov scaling regime remains stable under influence of helicity for all values of the helicity parameter.

The correctness of this result is confirmed by two independent methods of calculations of the corresponding two-loop Feynman diagrams. By comparison of these two methods the analytic expressions for two classes of definite integrals are found (see Appendix C).

The fact that helicity does not destroy the stability of the Kolmogorov scaling regime will be used in further investigations of the situations where helicity can play essential role, namely, in the two-loop investigation of the turbulent dynamo in the helical turbulent magnetohydrodynamics, in the two-loop investigation of the influence of the helicity on the anomalous scaling of passive scalar advected by the velocity field driven by the stochastic Navier-Stokes equation, in the calculation of the influence of helicity on the inverse Prandtl number of the same problem, and, last but not least, it is also the first necessary step for determination of the dependence of the Kolmogorov constant on the helicity parameter.

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APPENDIX A

The explicit form of the coefficients A_l, $B_{li}(i=0,1,2)$, and $C_{lj}(j=1,2)$ for $l=1,\ldots,8$ in Eqs. (31)–(33) is the following:

$$A_1 = \frac{(d-2)(d+1)}{4}, \quad A_2 = \frac{d^2-d+1}{2},$$

$$A_3 = \frac{d(d-1)}{4}, \quad A_4 = 0,$$

$$A_5 = -A_7 = \frac{1}{2}, \quad A_6 = -A_8 = -1,$$

$$B_{10} = \frac{x^2}{2}\left[8x^2 - 6 - \frac{d(x^2-2) - 2x^4 + 9x^2 - 8}{4-x^2}\right],$$

$$B_{11} = x\sqrt{1-x^2}[4(d-2)(d+1)x^2 - d^2 + d - 2]/2,$$

$$B_{12} = -x\{-16[2+(d-1)d] + 4\{d[d(7+d)-11]-2\}x^2 - \{d[d(14+d)-17]-22\}x^4 + 2(d-2)(d+1)x^6\}/[4(4-x^2)^{3/2}],$$

$$B_{20} = \{32 + 4[-1+2d(1+d+d^2)]x^2 - \{23 + d[-9+2d(5+d)]\}x^4 + 2[2+(d-1)d]x^6\}/[4(4-x^2)^2],$$

$$B_{21} = \sqrt{1-x^2}[1 + x^2 + 4(d^2-d+1)x^4]/x,$$

$$B_{22} = \{-64 + 8x^2 - 2\{5 + d[-21 + 2d(13+5d)]\}x^4 + \{50 + d[-60 + d(61+9d)]\}x^6 - \{19 + d[-20 + d(20+d)]\}x^8 + 2[1+(d-1)d]x^{10}\}/[2x(4-x^2)^{5/2}],$$

$$B_{30} = 0,$$

$$B_{31} = x\sqrt{1-x^2}(4x^2-1)d(d-1)/2,$$

$$B_{32} = xd(d-1)[4 - (7+d)x^2 + 2x^4]/(4\sqrt{4-x^2}),$$

$$B_{40} = \{-64 - 8\{-17 + d[6 + d(3d-7)]\}x^2 + 2\{-37 + d[9 + d(3d-5)]\}x^2 + [2-(d-3)d]x^6\}/[4(1-x^2)(4-x^2)^2],$$

$$B_{41}=\{-8+(16+d-d^2)x^2+2[-4+(d-1)^2d]x^4\}/[4x(1-x^2)^{3/2}],$$

$$B_{42}=\{128+16[-9+(d-1)d]x^2-4\{-5+d[-3+(d-1)d]\}x^4+[-4+(d-1)^2d]x^6\}/[2x(4-x^2)^{5/2}],$$

$$B_{50}=\{32(d-1)+4[1+d(16d-63)]x^2-[-99+4d(17d-94)]x^4+[-58+d(21d-142)]x^6-2[-4+(d-8)d]x^8\}/[4(4-x^2)^2],$$

$$B_{51}=\{\sqrt{1-x^2}[2(d-1)+[-8+d(7d-15)]x^2-2[-12+d(3d-4)]x^4]\}/(2x),$$

$$B_{52}=\{-64(d-1)-8[-23+d(28d-69)]x^2+2(-407-450d+212d^2)x^4-2[-321+d(116d-263)]x^6 +[-176+d(51d-128)]x^8-[-16+d(4d-11)]x^{10}\}/[2x(4-x^2)^{5/2}],$$

$$B_{60}=x^2[-8+3x^2+d(10-11x^2+2x^4)]/[2(4-x^2)],$$

$$B_{61}=x^3\sqrt{1-x^2}(d^2+4d-4),$$

$$B_{62}=x^3\{-6[-6+d(3+4d)]+[-26+d(13+10d)]x^2-[-4+d(2+d)]x^4\}/[2(4-x^2)^{3/2}],$$

$$B_{70}=\{-96d+4\{16+d[75+2(d-1)d]\}x^2-\{124+d[291+2(d-15)d]\}x^4 -[-62+d(15d-86)]x^6+2[-4+(d-4)d]x^8\}/[4(4-x^2)^2],$$

$$B_{71}=\sqrt{1-x^2}[-6d+(1+d)(8+d)x^2+4(-4+d+d^2)x^4]/(2x),$$

$$B_{72}=\{192d-8[32+d(63+4d)]x^2-2\{-340+d[-191+2d(21+d)]\}x^4+\{-504+d[-76+d(99+d)]\}x^6 -(-134+d+31d^2)x^8+(-12+d+3d^2)x^{10}\}/[2x(4-x^2)^{5/2}],$$

$$B_{80}=\{384d+8[4+d(-51+20d)]x^2+2\{-2+d[-11+2d(3+d)]\}x^4-\{15+d[-268+d(41+d)]\}x^6 +[2+d(7d-108)]x^8+12dx^{10}\}/[4x^2(4-x^2)^2],$$

$$B_{81}=\{48d+[4+5d(4d-13)]x^2+\{-4+d[27+d(3d-17)]\}x^4-2d(14+3d)x^6+24dx^8\}/(4x^3\sqrt{1-x^2}),$$

$$B_{82}=\{-1536d-32[4+d(20d-71)]x^2-8\{-18+d[175+6d(2d-13)]\}x^4+4(-5+d(226+d(11d-19)))x^6 -(-4+d(507+d(34+5d)))x^8+6d(22+d)x^{10}-12dx^{12}\}/[4x^3(4-x^2)^{5/2}],$$

$$C_{11}=-[2(d+1)x^2+1]\sqrt{1-x^2},$$

$$C_{12}=\frac{(d+1)x^6-(7d+6)x^4+4(3d+1)x^2+8}{(4-x^2)^{3/2}},$$

$$C_{21}=-\frac{\sqrt{1-x^2}[2+4x^2+d(d-1)(1+4x^2)]}{2},$$

$$C_{22}=-\{-32(d^2-d+2)-4[3+14(d-1)d]x^2 +2[29+34(d-1)d]x^4-[20+21(d-1)d]x^6 +2(d^2-d+1)x^8\}/[2(4-x^2)^{5/2}],$$

$$C_{31}=-d(d-1)x^2\sqrt{1-x^2},$$

$$C_{32}=-\frac{d(d-1)x^2(x^2-3)}{2\sqrt{4-x^2}},$$

$$C_{41}=-\frac{d(d-1)(2x^2-1)}{2(1-x^2)^{3/2}},$$

$$C_{42}=-2[8d(d-1)-2(3d^2-3d-1)x^2 +(d^2-d+1)x^4]/(4-x^2)^{5/2},$$

$$C_{51}=-\frac{[2+(d-1)d+6x^2]\sqrt{1-x^2}}{2},$$

$$C_{52} = -\{-32(d^2-d+2)+4[4d(2d-5)-27]x^2 - 2[5(d-3)d-89]x^4+[(d-3)d-60]x^6 + 6x^8\}/[2(4-x^2)^{5/2}],$$

$$C_{61} = -\frac{(1+x^2)\sqrt{1-x^2}}{2},$$

$$C_{62} = \frac{[16+4(d-1)dx^2-(10+(d-1)d)x^4+2x^6]}{2(4-x^2)^{3/2}},$$

$$C_{71} = -(d+2x^2)\sqrt{1-x^2},$$

$$C_{72} = -\{-32d-4[15+(d-9)d]x^2+[70+(d-11)d]x^4 + (d-21)x^6+2x^8\}/[(4-x^2)^{5/2}],$$

$$C_{81} = \frac{d(d-3)(1+2x^2)}{4\sqrt{1-x^2}},$$

$$C_{82} = -\{16(d-3)d+4[1+3(d-3)d]x^2-2[4(d-3)d-1]x^4 + (d-3)dx^6\}/[2(4-x^2)^{5/2}],$$

APPENDIX B

In this appendix we present another analytical representation of the divergent helical parts $\Gamma_l^{2,\rho}$ of the two-loop diagrams which are shown graphically in Fig. 3. They can be obtained by using the technique introduced in Appendix A in Ref. [37]. The results are

$$\Gamma_l^{2,\rho} = -\frac{S_d^2}{(2\pi)^{2d}}\frac{g^2\nu p^2\delta_{ij}}{16\varepsilon}\left(\frac{\mu}{m}\right)^{4\varepsilon}\frac{\pi(d-2)}{d^2(d+2)}D_l, \qquad \text{(B1)}$$

where coefficients D_l, $l=1,\ldots,8$ are defined as follows:

$$D_1 = (-4H_0+dH_1+H_2)/8,$$

$$D_2 = \{12(d-1)^2(d-2)H_0 - d\{-4+d[25+d(-19+6d)]\}H_1 - (d-1)^2(4d-5)H_2\}/[48(d+1)],$$

$$D_3 = d(d-1)(-8H_0+3dH_1+2H_2)/[16(d+1)],$$

$$D_4 = \{-6(d-1)^3H_0+(d-2)\{d[-2+d(-2+3d)]H_1 - (d-1)^2H_2\}\}/[24(d-2)],$$

$$D_5 = (24(-4+d-d^3)H_0-(-22 + d\{13+d[-29+d(11+3d)]\})H_2-d\{-68 + d[23+d(-8+9d)]\}H_1)/[96(d+1)],$$

$$D_6 = d[8H_0+2(1-2d)H_1+(d^2-5)H_2]/[16(d+1)],$$

$$D_7 = \{48(d^2+d-2)H_0+d[-37+d(-14+3d)]\}H_1-(d-1) \times[20+d(3+d)(1+3d)]\,H_2)/[96(d+1)],$$

$$D_8 = \{-12(d-3)^2H_0+d[-4+d(-7+3d)]H_1 + \{-2+d[31+d(-20+3d)]\}H_2\}/96,$$

where H_0, H_1, and H_2 are the following functions:

$$H_0 = \frac{\Gamma\left(1+\frac{d}{2}\right)^2}{\Gamma\left(\frac{1+d}{2}\right)^2},$$

$$H_1 = {}_2F_1\left(\frac{1}{2},\frac{1}{2};\frac{d}{2};\frac{1}{4}\right),$$

$$H_2 = {}_2F_1\left(\frac{1}{2},\frac{1}{2};1+\frac{d}{2};\frac{1}{4}\right),$$

where ${}_2F_1(a,b;c;z)$ is the corresponding hypergeometric function.

APPENDIX C

We have two nonequivalent representations of the divergent helical parts of the two-loop diagrams shown in Fig. 3, namely, the integral representation given in Eq. (31) by the part proportional to ρ^2 with coefficients C_l given in Appendix A and the representation given in Eq. (B1) in Appendix B with coefficients D_l which is in the form of combinations of the hypergeometric functions. Simple numerical comparison of these representations leads to the following general result:

$$\int_0^1 dx\frac{(1-x^2)^{(d-1)/2}}{x^2}C_l = \frac{d-1}{d}\frac{S_d}{S_{d-1}}\pi D_l. \qquad \text{(C1)}$$

Besides, during the calculations also the following results for definite integrals were obtained

$$\int_{-1}^1 dx(1-x^2)^{(d-4)/2}x^{2n}\arctan\left(\frac{1+x}{\sqrt{1-x^2}}\right) = \frac{(2n-1)!!\pi^{3/2}}{2^{n+2}}\frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d+2n-1}{2}\right)},$$

for $n\geq 0$ and $d>2$, and

$$\int_{-1}^1 dx(1-x^2)^{(d-3)/2}\frac{x^{2n}}{\sqrt{4-x^2}}\arctan\left(\frac{1+x}{\sqrt{1-x^2}}\right) = \frac{(2n-1)!!\pi^{3/2}}{2^{n+3}}\frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}+n\right)}{}_2F_1\left(\frac{1}{2},\frac{2n+1}{2};n+\frac{d}{2},\frac{1}{4}\right),$$

for $n\geq 0$ and $d>1$. Here, it is considered that $(-1)!!=1$ by definition.

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